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Numerival codes, i.e. Autodyn are widely used for simulations purpose during impact and penetration of prosjectiles into targets. To check the validity of the simulations during shock compression we first study the axiomatic structure of the analytical shock mechanics mor slosely. Thereafter the analytical results are compared with results from the simulations furing a onedimensional planar situation. By comparing the analytical shock solutions with the Autodyn solutions, we find that the density, pressure and shock velocity are in good agreement with the results from the analytical theory. But, we find that the energy at the reat of the shock show deviations from the analutivla rwatults. The reasjon for this is probably related to the use of artificial viscosity inherent in the Autodyn codes. The use of the Autodyn hydro code for shock impact problems where the temperature or the energy is important can therefore be questioned.

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CONTENTS

COMPARISON BETWEEN ANALYTICAL SHOCK MECHANIC SOLUTIONS AND NUMERICAL SOLUTIONS FROM THE AUTODYN HYDRO CODE

1 INTRODUCTION

Numerical codes, i.e. Autodyn[1] are widely used for simulations purposes during impact and penetration of projectiles into targets. To check the validity of the simulations during shock compression we first study the axiomatic structure of the analytical shock mechanics more closely. Thereafter the analytical results are compared with results from the simulations during a one-dimensional planar situation. By comparing the analytical shock solutions with the Autodyn solutions, we find that the density, pressure and shock velocity are in good agreement with the results from the analytical theory. The energy in the shock shows deviations from the analytical results. The reason for this is probably related to the use of artificial viscosity inherent in the Autodyn code since by using different values for the viscosity coefficients we achieve different results for the energy.

We present the analytical shock theory more intuitively than usually seen in the literature. In appendix we provide a more rigorous mathematical presentation.

The discrepancy between the analytical and numerical solution for the shock energy, show that carefulness is necessary when using the Autodyn hydro code for shock impact problems.

2 THE CONSERVATION LAWS

The conservation laws includes three different equations; i) The conservation of mass, ii) the conservation of momentum and iii) the conservation of energy. These three equations will be analysed more closely in the following sections.

2.1 What is a shock?

Consider a situation where a sharp pulse is moving in a given frame of reference. A shock can be considered as:

i) A sharp rise in the pressure (or other variables) that cannot be described within a chosen theory of continuum mechanics due to grid resolution. The continuum model includes viscosity and heat conduction.

To study the shock phenomenon two different approaches are presented in the literature:

i) An analytical mathematical shock theory is proposed to describe the shock phenomenon.
ii) The continuum model is changed by adding large artificial viscosity.

We will in this section present the fundamental equations for the analytical shock model for approach i). Autodyn is believed to present approach ii).

2.2 Mass conservation

Consider a long tube where a rigid piston is moving with constant velocity ν.

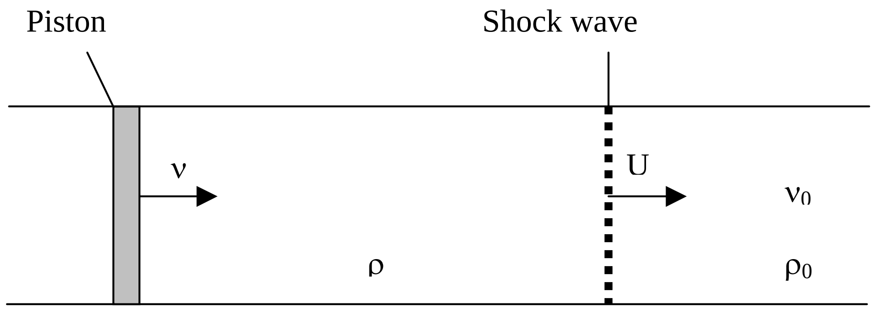

Figure 2.1: Piston moving along a long tube.

Let U be the velocity of the shock moving in front of the piston. Let ρ_0 be the density in front of the shock. Let ν_0 be the velocity in front of the shock. Finally let ρ be the density behind the shock. During a short time interval Δt the mass passing into the shock and out of the shock is given by

$$m_{out} = \rho\left(U-v\right)A\Delta t, m_{inn} = \rho_0\left(U-v_0\right)A\Delta t \qquad (2.2.1)$$

where A is the frontal area of the shock. Mass conservation gives that

$$m_{inn} = m_{out}, \Rightarrow \rho_0\left(U-v_0\right) \overset{mod}{=} \rho\left(U-v\right), \qquad (2.2.2)$$

where mod means model assumption. Equation (2.2.2) can easily be rearranged to the following different relations

$$U = \frac{v - v_0\left(\frac{\rho_0}{\rho}\right)}{1-\frac{\rho_0}{\rho}}, (a), U - v_0 = \frac{v-v_0}{1-\frac{\rho_0}{\rho}}, (b), \frac{\rho}{\rho_0} - 1 = \frac{v-v_0}{\left(U-v\right)}, (c), v - v_0 = \left(1-\frac{\rho_0}{\rho}\right)\left(U-v_0\right), (d) \qquad (2.2.3)$$

Since the density behind the shock is larger than the density on front of the shock it follows directly from (2.2.3d) that $U \geq v$. Thus the shock is running ahead and faster than the piston.

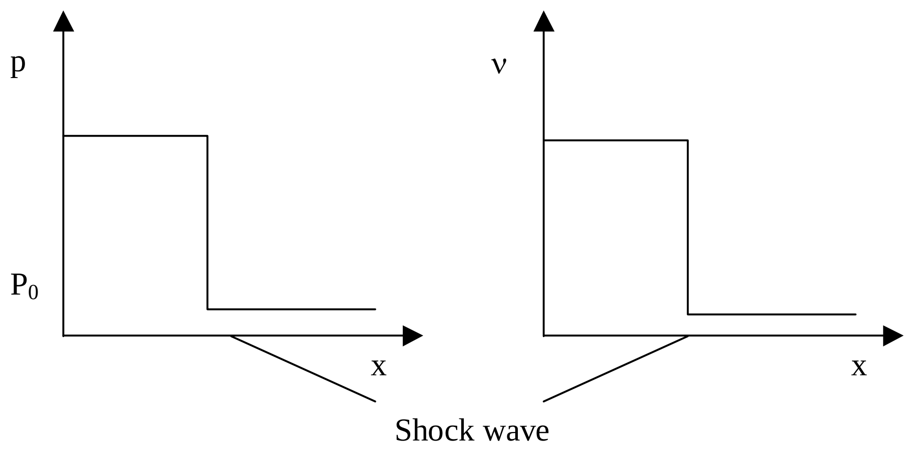

Figure 2.2: The pressure and velocity along the tube.

2.3 The momentum conservation

The momentum conservation through a shock is given as

$$-\left(p-p_0\right)A\Delta t \overset{mod}{=} \left(m_{out}(U-v)-m_{inn}(U-v_0)\right) \tag{2.3.1}$$

Inserting the relation for the masses given in (2.2.1) gives directly from (2.3.1) that

$$\begin{aligned} p-p_0 &= \rho_0\left(U-v_0\right)(U-v_0)-\rho\left(U-v\right)(U-v) \\ &= \rho v\left(U-v\right)-\rho_0 v_0\left(U-v_0\right)+\rho_0 U(U-v_0)-\rho U(U-v) \\ &= \rho v\left(U-v\right)-\rho_0 v_0\left(U-v_0\right) \end{aligned} \tag{2.3.2}$$

Further we can easily transform (2.3.2) by using (2.2.2) to the following equations

$$\begin{aligned} & p_0+\rho_0\left(U-v_0\right)^2 = p+\rho\left(U-v\right)^2,(a) \\ & p-p_0 = \rho_0\left(U-v_0\right)\left(v-v_0\right) = \frac{\rho_0\left(v-v_0\right)^2}{1-\rho_0/\rho} = \rho_0\left(U-v_0\right)^2\left(1-\rho_0/\rho\right),(b) \\ & \left(U-v_0\right) = \left(\frac{p-p_0}{\rho_0\left(1-\rho_0/\rho\right)}\right)^{1/2},(c), v-v_0 = \left(\left(p-p_0\right)\left(1-\rho_0/\rho\right)/\rho_0\right)^{1/2},(d) \end{aligned} \tag{2.3.3}$$

If the velocity and the pressure in front of the shock are zero (2.3.3) gives the well known equation

$$p = \rho_0 U v = \frac{\rho_0 v^2}{1-\rho_0/\rho} = \rho_0 U^2\left(1-\rho_0/\rho\right) \tag{2.3.4}$$

Thus the pressure behind the shock is given as the density in front of the shock times the velocity behind the shock times the shock velocity.

Observe that in general the pressure behind the shock is given when all quantities are given in front of the shock together with the velocity behind the shock and the shock velocity. By using the equation of state for the material, only one quantity is needed behind the shock to solve the shock equations completely. Further assume

$$p = p_0 + \Delta p, \rho = \rho_0 + \Delta\rho \tag{2.3.5}$$

Inserting (2.3.5) into (2.3.3c) gives that

$$\begin{aligned}(U - v_0) &= \left(\frac{\Delta p}{\rho_0 \left(1 - \rho_0 / (\rho_0 + \Delta\rho)\right)} \right)^{1/2} = \left(\frac{\Delta p}{\rho_0 \left(1 - 1/(1 + \Delta\rho / \rho_0)\right)} \right)^{1/2} \\ &= \left(\frac{\Delta p}{\rho_0 \left(1 - \left(1 - \Delta\rho / \rho_0 + (\Delta\rho / \rho_0)^2 + ..\right)\right)} \right)^{1/2} = \left(\frac{\Delta p}{\Delta\rho (1 - \Delta\rho / \rho_0 + ..)} \right)^{1/2} \\ &= \left(\frac{\Delta p}{\Delta\rho} \right)^{1/2} \left(1 + \Delta\rho / (2\rho_0) + ..\right)\end{aligned} \tag{2.3.6}$$

Thus to lowest order the shock velocity is equal with the shock adiabatic sound speed. More generally we have that

$$U^2(\rho) = \left(\frac{\rho}{\rho_0} \right) \frac{p(\rho)}{(\rho - \rho_0)} \geq \frac{\partial p(\rho)}{\partial \rho} \tag{2.3.7}$$

Further, observe that if the equation of state is independent of the temperature, the density behind the shock is given directly from (2.3.3d) by solving for the density when the velocity is given. If for example $p(\rho) = K(\rho / \rho_0 - 1)$, it follows from (2.3.3d) when $v_0 = p_0 = 0$ that

$$v^2 = \frac{p(\rho)\left(\frac{\rho}{\rho_0}-1\right)}{\rho} = \frac{K\left(\frac{\rho}{\rho_0}-1\right)^2}{\rho} \Rightarrow \left(\frac{\rho}{\rho_0}\right)^2 - 2\lambda\left(\frac{\rho}{\rho_0}\right)+1=0, \Rightarrow \left(\frac{\rho}{\rho_0}\right) = \lambda \pm \left(\lambda^2-1\right)^{1/2}, (a)$$

$$\lambda \overset{def}{=} \left(1+\frac{v^2\rho_0}{2K}\right),$$

$$\left(\frac{\rho}{\rho_0}\right) = 1+\left(v^2\rho/K\right)^{1/2} = 1+\frac{v}{\left(K/\rho_0\right)^{1/2}}+\ldots \quad , \frac{\rho}{\rho_0}-1 << 1,$$

$$\left(\frac{\rho}{\rho_0}\right) \approx 2+\frac{v^2\rho_0}{K}+\ldots \quad , \frac{\rho}{\rho_0}-1 >> 1,$$

(2.3.8)

To check the simulation results we first apply the linear equation of state for different bulk modulus. For a given piston velocity the density is found by solving (2.3.8a). The pressure is found by inserting the density and velocity into equation (2.3.3b) (or the equation of state can be used directly). Finally the shock velocity is found by inserting into (2.3.3c). Thus

$$\begin{aligned}
&A)\\
&\rho_0 = 1.50\cdot10^3 kg/m^3, K = 7.50\cdot10^8 Pa,\\
&v_{sim} = 1000 m/s, \rho_{sim} = 5.60\cdot10^3 kg/m^3, p_{sim} = 2.04\cdot10^9 Pa, U_{sim} = 1360 m/s,\\
&v_{theo} = 1000 m/s, \rho_{theo} = 5.59\cdot10^3 kg/m^3, p_{theo} = 2.05\cdot10^9 Pa, U_{theo} = 1370 m/s\\
&B)\\
&\rho_0 = 1.50\cdot10^3 kg/m^3, K = 7.50\cdot10^9 Pa,\\
&v_{sim} = 1000 m/s, \rho_{sim} = 2.34\cdot10^3 kg/m^3, p_{sim} = 4.17\cdot10^9 Pa, U_{sim} = 2800 m/s,\\
&v_{theo} = 1000 m/s, \rho_{theo} = 2.34\cdot10^3 kg/m^3, p_{theo} = 4.19\cdot10^9 Pa, U_{theo} = 2790 m/s\\
&C)\\
&\rho_0 = 1.50\cdot10^3 kg/m^3, K = 7.50\cdot10^7 Pa,\\
&v_{sim} = 1000 m/s, \rho_{sim} = 3.27\cdot10^4 kg/m^3, p_{sim} = 1.56\cdot10^9 Pa, U_{sim} = 1040 m/s,\\
&v_{theo} = 1000 m/s, \rho_{theo} = 3.29\cdot10^4 kg/m^3, p_{theo} = 1.57\cdot10^9 Pa, U_{theo} = 1050 m/s
\end{aligned} \quad (2.3.9)$$

Observe the close agreement between the simulations results from Autodyn and the theoretical analytical theory.

Further, adding a second order term to the equation of state we achieve that

$$p(\rho) = a_1(\rho/\rho_0 - 1) + a_2(\rho/\rho_0 - 1)^2, K = a_1$$

$$v^2 = \frac{p(\rho)\left(\frac{\rho}{\rho_0} - 1\right)}{\rho} = \frac{\left(a_1(\rho/\rho_0 - 1) + a_2(\rho/\rho_0 - 1)^2\right)\left(\frac{\rho}{\rho_0} - 1\right)}{\rho} \Rightarrow \tag{2.3.10}$$

$$\left(\frac{\rho}{\rho_0}\right)^3 + \left(\frac{a_1}{a_2} - 3\right)\left(\frac{\rho}{\rho_0}\right)^2 + \left(-\frac{2a_1}{a_2} + 3 - \frac{\rho_0 v^2}{a_2}\right)\left(\frac{\rho}{\rho_0}\right) + \left(\frac{a_1}{a_2} - 1\right) = 0$$

Solving this third order equation for ρ/ρ_0 and inserting into the shock velocity and pressure relation gives that

$$\begin{aligned}
&A)\\
&\rho_0 = 1.50 \cdot 10^3\, kg/m^3, a_1 = 7.50 \cdot 10^8\, Pa,\ a_2 = 7.50 \cdot 10^8\, Pa,\\
&v_{sim} = 1000 m/s, \rho_{sim} = 3.62 \cdot 10^3\, kg/m^3, p_{sim} = 2.56 \cdot 10^9\, Pa, U_{sim} = 1700 m/s\\
&v_{theo} = 1000 m/s, \rho_{theo} = 3.62 \cdot 10^3\, kg/m^3, p_{theo} = 2.56 \cdot 10^9\, Pa, U_{theo} = 1710 m/s\\
&B)\\
&\rho_0 = 1.50 \cdot 10^3\, kg/m^3, a_1 = 7.50 \cdot 10^9\, Pa,\ a_2 = 7.50 \cdot 10^8\, Pa,\\
&v_{sim} = 1000 m/s, \rho_{sim} = 2.31 \cdot 10^3\, kg/m^3, p_{sim} = 4.27 \cdot 10^9\, Pa, U_{sim} = 2840 m/s\\
&v_{theo} = 1000 m/s, \rho_{theo} = 2.31 \cdot 10^3\, kg/m^3, p_{theo} = 4.27 \cdot 10^9\, Pa, U_{theo} = 2850 m/s
\end{aligned} \tag{2.3.11}$$

Again the agreement between the numerical and analytical solutions is very good.

Assume that for some reason the experimental results give

$$U = c_1 + s_1 v \tag{2.3.12}$$

where c_1 and s_1 are two constants. The shock adiabatic relation is now easily found. First using (2.3.12) and (2.2.3b) gives

$$U = c_1 + s_1 v = \frac{v}{1 - \frac{\rho_0}{\rho}} \Rightarrow v = \frac{c_1\left(1 - \frac{\rho_0}{\rho}\right)}{\left(1 - s_1(1 - \frac{\rho_0}{\rho})\right)}, (a) \tag{2.3.13}$$

Thus from (2.3.4) when inserting (2.3.13a)

$$p = \frac{\rho_0 v^2}{1 - \rho_0/\rho} = \frac{\rho_0 {c_1}^2\left(1 - \frac{\rho_0}{\rho}\right)}{\left(1 - s_1(1 - \frac{\rho_0}{\rho})\right)^2} \tag{2.3.14}$$

Observe that the pressure reaches infinity for the density

$$\rho_{max} = \rho_0 \frac{s_1}{s_1 - 1} \tag{2.3.15}$$

Our final conclusion in this section is that Autodyn gives good results for the pressure, density and shock velocity during a shock. In the next section we analyse the internal energy.

2.4 The energy conservation

Stating a kind of Bernoulli equation most easily shows the energy conservation. Thus we have that

$$e_0 + \frac{p_0}{\rho_0} + \frac{1}{2}(U - v_0)^2 + q_0 = e + \frac{p}{\rho} + \frac{1}{2}(U - v)^2 \tag{2.4.1}$$

e_0 and e is the internal energy of the substance before and after the shock. q_0 is the energy release during a shock due to a chemical decomposition. If $v_0 = p_0 = 0$ we have from (2.4.1) that

$$e_0 = e + \frac{p}{\rho} - vU + \frac{1}{2}v^2 - q_0 = e + v(U - v) - vU + v^2 - q_0 = e - \frac{1}{2}v^2 - q_0 \tag{2.4.2}$$

Thus

$$e - e_0 = \frac{1}{2}v^2 + q_0 \tag{2.4.3}$$

This equation simply says that the energy at the rear of the shock equals the kinetic energy of the particle plus a term related to the chemical decomposition. By using (2.3.8) we also have that

$$e - e_0 = \frac{1}{2}v^2 + q_0 = \frac{p(\rho)\left(\frac{\rho}{\rho_0} - 1\right)}{2\rho} + q_0 \tag{2.4.4}$$

The "shock energy" can be compared with the adiabatic energy given by

$$e(T, \rho) - e_0 = \int_{\rho_0}^{\rho} \frac{p(T(\rho'), \rho')}{\rho'^2} d\rho' + q_0,\ T(\rho_0) = T_0 \tag{2.4.5}$$

For a given equation of state this gives a special solution for $T(\rho')$;the adiabatic.

As an example consider the simple relation $e(T,\rho)=e_0+c_v(T-T_0), q_0=0$. Inserting into (2.4.5) gives for the Poisson adiabatic (a) and the shock adiabatic (b) for the linear equation of state

$$\Delta e'_{theo}=c_v\cdot(T-T_0)=K\int_{\rho_0}^{\rho}\left(\frac{1}{\rho'\rho_0}-\frac{1}{\rho'^2}\right)d\rho'=\left(\frac{K}{\rho_0}\right)\left(Log\left(\rho/\rho_0\right)+\rho_0/\rho-1\right),(a)$$

$$\Delta e''_{theo}=c_v\cdot(T-T_0)=\frac{1}{2}v^2=\left(\frac{K}{\rho_0}\right)\frac{\rho_0\left(\frac{\rho}{\rho_0}-1\right)^2}{2\rho},(b) \qquad (2.4.6)$$

Comparing simulations results with theoretical values we achieve for the linear equation of state that

$$\begin{aligned}
&A)\Delta e_{sim}=5.82\cdot10^5\,J/kg,\Delta e'_{theo}=2.92\cdot10^5\,J/kg,\Delta e''_{theo}=5.00\cdot10^5\,J/kg\\
&B)\Delta e_{sim}=5.20\cdot10^5\,J/kg,\Delta e'_{theo}=4.26\cdot10^5\,J/kg,\Delta e''_{theo}=5.00\cdot10^5\,J/kg \qquad (2.4.7)\\
&C)\Delta e_{sim}=5.69\cdot10^5\,J/kg,\Delta e'_{theo}=1.06\cdot10^5\,J/kg,\Delta e''_{theo}=5.00\cdot10^5\,J/kg
\end{aligned}$$

Observe that the theoretical results for the energy deviate from the numerical results. Also observe that the “shock energy” is larger than the adiabatic energy. The reason for this is the well-known fact that during a shock some amount of energy is released due to shear stresses.

As the second example we again consider the non linear equation of state. Using (2.3.8), (2.3.4)-(2.3.5) gives for the shock adiabatic and the adiabatic that

$$\Delta e'_{theo}=\frac{a_1}{\rho_0}Log(\rho/\rho_0)+a_1\left(\frac{1}{\rho}-\frac{1}{\rho_0}\right)+\frac{a_2}{\rho_0^2}(\rho-\rho_0)-\frac{2a_2}{\rho_0}Log(\rho/\rho_0)-a_2\left(\frac{1}{\rho}-\frac{1}{\rho_0}\right),(a)$$

$$\Delta e''_{theo}=\frac{a_1\left(\frac{\rho}{\rho_0}-1\right)^2+a_2\left(\frac{\rho}{\rho_0}-1\right)^3}{2\rho},(b)$$

$$(2.4.8)$$

Thus we achieve for the non linear equation of state

$$\begin{aligned}
&A)\Delta e_{sim}=6.00\cdot10^5\,J/kg,\Delta e'_{theo}=2.66\cdot10^5\,J/kg,\Delta e''_{theo}=5.00\cdot10^5\,J/kg\\
&B)\Delta e_{sim}=5.22\cdot10^5\,J/kg,\Delta e'_{theo}=4.19\cdot10^5\,J/kg,\Delta e''_{theo}=5.00\cdot10^5\,J/kg \qquad (2.4.9)
\end{aligned}$$

Observe that the theoretical results for the energy again deviate from the numerical results.

We believe that the reason for the unphysical simulations results is related to the artificial viscosity approach inherent in the code. To test this we have intentionally increased the two-viscosity coefficient from their standard values. The linear equation of state was used for all cases.

In the hydro code Autodyn the standard values for the two viscosity coefficients are:
$Linear: \mu_1 = 0.2,\ Quadratic: \mu_2 = 1.0$

For case A, linear equation of state, we tested:
1) $Linear: \mu_1 = 0.2,\ Quadratic: \mu_2 = 10.0$ (maximum value)
And
2) $Linear: \mu_1 = 10.0$ (maximum value), $Quadratic: \mu_2 = 1.0$

When we increased μ_2 to 10.0 the internal energy, Δe_{sim}, varied from $5.95 \cdot 10^5 J/kg$ (close to the impact) to $5.75 \cdot 10^5 J/kg$ (longer away).

When we increased μ_1 to 10.0 the internal energy, Δe_{sim}, varied in the following way: When the shock travelled in the material the internal energy jumped to $5.90 \cdot 10^5 J/kg$ (close to the impact) and $6.25 \cdot 10^5 J/kg$ (longer away) but increased linearly with time.

For density, pressure and shock-velocity we achieve that:

$A, 1)$
$\rho_0 = 1.50 \cdot 10^3 kg/m^3, K = 7.50 \cdot 10^8 Pa,$
$v_{sim} = 1000 m/s, \rho_{sim} = 5.60 \cdot 10^3 kg/m^3, p_{sim} = 2.04 \cdot 10^9 Pa, U_{sim} = 1370 m/s,$
$v_{theo} = 1000 m/s, \rho_{theo} = 5.59 \cdot 10^3 kg/m^3, p_{theo} = 2.05 \cdot 10^9 Pa, U_{theo} = 1370 m/s$

$A, 2)$
$\rho_0 = 1.50 \cdot 10^3 kg/m^3, K = 7.50 \cdot 10^8 Pa,$
$v_{sim} = 1000 m/s, \rho_{sim} = 5.00 \cdot 10^3 kg/m^3, p_{sim} = 1.75 \cdot 10^9 Pa, U_{sim} = 1440 m/s,$
$v_{theo} = 1000 m/s, \rho_{theo} = 5.59 \cdot 10^3 kg/m^3, p_{theo} = 2.05 \cdot 10^9 Pa, U_{theo} = 1370 m/s$

Thus for case A,2) also the pressure and the density deviates from the analytical solution.

3 CONCLUSIONS

By comparing the analytical shock solutions with the Autodyn solutions, we find that the density, pressure and shock velocity are in good agreement with the results from the analytical theory. But, we find that the energy at the rear of the shock show large deviations from the analytical results. The reason for this is probably related to the use of artificial viscosity inherent in the Autodyn code. Increasing intentionally the linear artificial viscosity coefficient to large values also influences the density and pressure behind the shock front. Carefulness must be addressed when using Autodyn hydro code for shock impact problems where the temperature or the energy is important for the solution.

References

(1) AUTODYN, Theory Manual, Revision 3.0

APPENDIX A; THE MASS CONSERVATION

The mass conservation is on tensor from written as

$$\partial\rho/\partial t+\partial_i\left(\rho v_i\right)=0 \tag{A1}$$

Thus applying the standard rule

$$\partial/\partial t\rightarrow -U[\ \ ],\partial/\partial x\rightarrow[\ \ ],[\ \ ]\ \textit{means step} \tag{A2}$$

gives for a planar one dimensional situation

$$-U[\rho]+[\rho v]=0\Leftrightarrow -U\left(\rho_1-\rho_2\right)=-\left(\rho_1 v_1-\rho_2 v_2\right)\Leftrightarrow \rho_1\left(U-v_1\right)=\rho_2\left(U-v_2\right) \tag{A3}$$

where the subscript 1 denotes in front of the shock and the subscript 2 denotes behind the shock.

APPENDIX B; THE MOMENTUM CONSERVATION

The equation of motion can be written as

$$\rho\left(\partial v_i/\partial t+v_j\partial_j v_i\right)=\partial_j\sigma_{ij},\sigma_{ij}=-p\delta_{ij}+s_{ij} \tag{B1}$$

From (A1) and (B1) it follows directly that

$$\frac{\partial\left(\rho v_i\right)}{\partial t}+\partial_j\left(\rho v_i v_j-\sigma_{ij}\right)=0 \tag{B2}$$

Using the rule in (A2) gives that

$$\begin{gathered}-U[\rho v]+\left[\rho v^2+p\right]=0\Leftrightarrow -U\left(\rho_1 v_1-\rho_2 v_2\right)+\left(\rho_1 v_1{}^2+p_1-\rho_2 v_2{}^2-p_2\right)=0,\\ \sigma=-p,[s]=0\end{gathered} \tag{B3}$$

Equation (B3) can also be written as

$$p_2 - p_1 = \rho_2 v_2 \left(U - v_2\right) - \rho_1 v_1 \left(U - v_1\right) \tag{B4}$$

Inserting (A3) in (B4) gives that

$$p_2 - p_1 = v_2 \rho_1 \left(U - v_1\right) - v_1 \rho_1 \left(U - v_1\right) = \rho_1 \left(v_2 - v_1\right)\left(U - v_1\right), \tag{B5}$$

Observe that the pressure behind the shock is given when all quantities are given in front of the shock together with the velocity behind the shock and the shock velocity.

APPENDIX C; THE ENERGY CONSERVATION

The energy equation can be written as

$$\rho\left(\partial e / \partial t + v_j \partial_j e\right) = \sigma_{ij} \partial_j v_i + \kappa \partial_i \partial_i T + q \tag{C1}$$

e is the internal energy, T is the temperature and q is a energy generating source. From (A1), (B1) and (C1) is it easy to verify that

$$\partial\left(\frac{1}{2}\rho v_i^{\,2} + \rho e\right) / \partial t + \partial_j \left(\left(\frac{1}{2}\rho v_i^{\,2} + \rho e\right) v_j - \sigma_{ij} v_i - \kappa \partial_j T + q\right) = 0 \tag{C2}$$

Applying the standard rule in (A2) gives from (C2) that

$$-U\left[\frac{1}{2}\rho v^2 + \rho e\right] + \left[\left(\frac{1}{2}\rho v^2 + \rho e\right) v - p v + q\right] = 0, \left[\partial T\right] = 0 \tag{C3}$$

Thus

$$-U\left(\frac{1}{2}\rho_1 v_1{}^2 + \rho_1 e_1 - \frac{1}{2}\rho_2 v_2{}^2 - \rho_2 e_2\right)$$

$$+\frac{1}{2}\rho_1 v_1{}^2 v_1 + \rho_1 e_1 v_1 - \frac{1}{2}\rho_2 v_2{}^2 v_2 - \rho_2 e_2 v_2 + p_1 v_1 - p_2 v_2 + q_1 - q_2 = 0,$$

$$= \rho_1 e_1 (v_1 - U) - \rho_2 e_2 (v_2 - U) + \frac{1}{2}\rho_1 v_1{}^2 (v_1 - U) - \frac{1}{2}\rho_2 v_2{}^2 (v_2 - U) \quad \text{(C4)}$$

$$+ p_1 (v_1 - U) - p_2 (v_2 - U) - U\left(p_2 - p_1\right)$$

$$= \rho_1 e_1 (v_1 - U) - \rho_2 e_2 (v_2 - U) + \frac{1}{2}\rho_1 v_1{}^2 (v_1 - U) - \frac{1}{2}\rho_2 v_2{}^2 (v_2 - U)$$

$$+ p_1 (v_1 - U) - p_2 (v_2 - U) + U\left(\rho_2 v_2 \left(v_2 - U\right) - \rho_1 v_1 \left(v_1 - U\right)\right) + q_1 - q_2 = 0,$$

By inserting using the equation of mass conservation in (A3) into (C4), we reach after dividing with $\rho_1 (v_1 - U)$

$$\begin{pmatrix} \rho_1 e_1 (v_1 - U) - \rho_2 e_2 (v_2 - U) + \frac{1}{2}\rho_1 v_1{}^2 (v_1 - U) - \frac{1}{2}\rho_2 v_2{}^2 (v_2 - U) \\ + p_1 (v_1 - U) - p_2 (v_2 - U) + U\left(\rho_2 v_2 \left(v_2 - U\right) - \rho_1 v_1 \left(v_1 - U\right)\right) + q_1 - q_2 \end{pmatrix} / \left(\rho_1 (U - v_1)\right)$$

$$= e_1 + p_1/\rho_1 + \frac{1}{2}(U - v_1)^2 + q_1/\left(\rho_1 (U - v_1)\right) - \left(e_2 + p_2/\rho_2 + \frac{1}{2}(U - v_2)^2 + q_2/\left(\rho_2 (U - v_2)\right)\right) = 0$$

(C5)

APPENDIX D; NUMERICAL GRIDS

In the simulations two numerical grids were used. A Lagrange grid (1000 mm length and 100 width mm) is moved into a Euler grid (1000 mm length 100 width) with a constant axial velocity of 1000 m/s for all times. The Lagrangian grid is thus un-deformed. The Lagrange grid was divided into 251*26 numerical cells and the Euler grid was divided into 1001*101 numerical cells. Figure D1 shows a cross-section of the numerical setup. For post processing the results a number of 100 “Target points” were put into the Euler grid along the centre line of the grid. The different target points show approximately the same values.

The material in the Lagrange grid was steel. In the Euler grid the following values were applied (Case 1):

```
Equation of state              :  Linear/Polynomial
Reference density     (g/cm3):  1.50000E+00
Bulk Modulus          (kPa)  :  7.50000E+05
Reference Temperature (K)    :  2.73000E+02
Specific Heat (C.V.)  (J/kgK):  1.00000E+03
STRENGTH MODEL                :  None (Hydro)
FAILURE MODEL                 :  None
Erosion model                 :  None
```

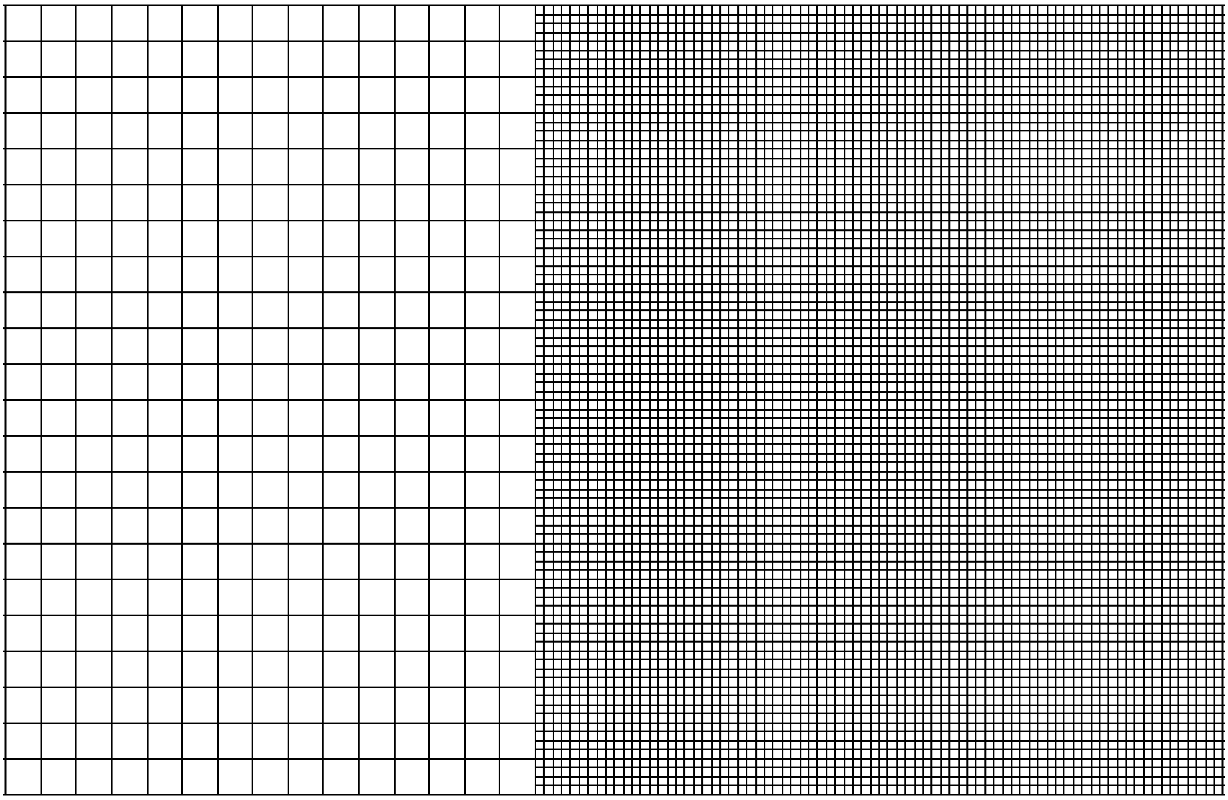

Figure D1: Cross-section of the numerical grids. Rigid Lagrange grid with velocity 1000m/s.,